

# PECULIARITIES OF THE LIGHT ABSORPTION AND EMISSION BY FREE ELECTRONS IN MULTIVALLEY SEMICONDUCTORS

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General expressions are obtained for the coefficient of light absorption by free carriers as well as the intensity of the spontaneous light emission by hot electrons in multivalley semiconductors. These expressions depend on the electron concentration and electron temperature in the individual valleys. An anisotropy of the dispersion law and electron scattering mechanisms is taken into account. Impurity-related and acoustic scattering mechanisms are analyzed. Polarization dependence of the spontaneous emission by hot electrons is found out. At unidirectional pressure applied or high irradiation intensities, the polarization dependence also appears in the coefficient of light absorption by free electrons.

heated (e.g., by a constant electric field), the process of light emission by free electrons occurs.

A new effect related to the light emission by free electrons in multivalley semiconductors, comparing to single-valley semiconductors, is the appearance of the polarization dependence of the emitted light intensity. The same dependence can also appear in absorption if the irradiation intensity of a multivalley semiconductor is high enough.

There are various methods that allow finding the absorption and emission by free carriers. In our opinion, the most convenient method is to use the kinetic equation, in which the influence of the electromagnetic field on the free carriers scattering mechanism is taken into account. The convenience of this method lies in that one may derive the expression for the absorption by free carriers both in classical and quantum cases in a single approach. Besides absorption, this method allows also finding the wave-field-induced emission by free carriers. From here, one may obtain the spontaneous emission by free carriers through definite formal substitutions. We have used such an approach in [1]. In that paper, we have investigated the mechanism of acoustic scattering in detail, and outlined a model for the impurity scattering. However, no analytic expressions for both classical and quantum absorption under the impurity scattering in multivalley semiconductors have been derived. The same is true also for the emission regularities. That is why we pay the principal attention in this paper to the study of the situations where the impurity scattering dominates.

## Introduction

The phenomena of the light absorption and emission by free carriers have been studied for years, and it seems likely that a discovery of new effects can hardly be expected here. However, this is not true in the case of multivalley semiconductors. The peculiarities of the mentioned phenomena in such semiconductors are related to (i) a sharp anisotropy of the dispersion law for electrons, (ii) the fact of the electron filling of several valleys and, finally, (iii) an anisotropy of scattering mechanisms. It is known that a “third body” is required in the act of the photon emission or absorption by a free electron. This “third body” provides the energy and momentum conservation during collisions. Impurity atoms, lattice oscillations (phonons), or boundaries can serve for it. By this, the influence of scattering mechanisms (including their anisotropy) on the absorption and emission processes can be explained.

In the thermodynamic equilibrium state, the quantity of photons absorbed by free electrons is equal to that of emitted photons. The detailed balancing principle is therefore valid. If the photon quantity exceeds that in equilibrium state, i. e., a semiconductor is irradiated with an external electromagnetic field, the photons are absorbed by free carriers. If the external electromagnetic field is absent, and the electron gas is

## 1. Collision Integral of Electrons with Ions in the Presence of an Electromagnetic Wave

We consider the multivalley semiconductors like  $n$ -Ge and  $n$ -Si. The Hamiltonian of electrons, which populate one of the conduction band valleys, can be written in the principal axes of the mass ellipsoid in the presence of an

electromagnetic field as follows:

$$\hat{H} = \sum_{\alpha=1}^3 \frac{1}{2m_{\alpha}} \left( \hat{p}_{\alpha} - \frac{e_0}{c} A_{\alpha} \right)^2 + \sum_{i=1}^N U(\vec{r} - \vec{R}_i). \quad (1)$$

In Eq. (1),  $m_{\alpha}$  are the principal components of the mass tensor, ( $m_x = m_y \equiv m_{\perp}$ ,  $m_z = m_{\parallel}$ ),  $\hat{p}_{\alpha}$  is the  $\alpha$ th component of the momentum operator,  $e_0$  is the electron charge,  $c$  is the light velocity,  $A_{\alpha}$  is the  $\alpha$ th component of the vector potential of electromagnetic field,  $N$  is the number of ions in volume  $V$ ,  $U(\vec{r} - \vec{R}_i)$  is the interaction potential of an electron with an ion ( $\vec{r}$  is the electron coordinate,  $\vec{R}_i$  is the coordinate of the  $i$ th ion),

$$U(\vec{r}) = \frac{e_0^2}{\varepsilon_0 r} e^{-r/r_D}, \quad (2)$$

$\varepsilon_0$  is the static dielectric constant, and  $r_D$  is the Debye radius.

We set the vector potential  $\vec{A}$  in a form

$$\vec{A} = \vec{A}^{(0)} \cos \omega t. \quad (3)$$

In formula (3),  $\vec{A}^{(0)}$  is a constant vector,  $\omega$  is the wave frequency.

The electron wave function in the field of an electromagnetic wave but without scattering centers, is determined from the Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \psi_{\vec{p}}^{(0)} = \hat{H}^{(0)} \psi_{\vec{p}}^{(0)} \equiv \sum_{\alpha=1}^3 \frac{1}{2m_{\alpha}} \left( \hat{p}_{\alpha} - \frac{e_0}{c} \vec{A}_{\alpha} \right)^2 \psi_{\vec{p}}^{(0)} \quad (4)$$

and equals:

$$\begin{aligned} \psi_{\vec{p}}^{(0)} &= \frac{1}{\sqrt{V}} \exp \left( \frac{i}{\hbar} \vec{p} \vec{r} \right) \times \\ &\times \exp \left\{ -\frac{i}{\hbar} \int_0^t dt' \sum_{\alpha=1}^3 \frac{1}{2m_{\alpha}} \left( p_{\alpha} - \frac{e_0}{c} \right) A_{\alpha}(t')^2 \right\} \approx \\ &\approx \frac{\exp \left( \frac{i}{\hbar} \vec{p} \vec{r} \right)}{\sqrt{V}} \exp \left\{ -\frac{i}{\hbar} \varepsilon_{\vec{p}} t + \frac{ie_0}{c\hbar\omega} \sum_{\alpha=1}^3 \frac{p_{\alpha} A_{\alpha}^{(0)}}{m_{\alpha}} \sin \omega t \right\}. \end{aligned} \quad (5)$$

In formula (5),  $V$  is the system volume,  $\varepsilon_{\vec{p}} = \sum_{\alpha=1}^3 p_{\alpha}^2 / 2m_{\alpha}$  is the energy of electron having momentum  $\vec{p}$ . We have omitted the quadratic components in  $A_{\alpha}^{(0)}$  in the exponent, when obtaining formula (5). We shall find the electron wave function in the presence of scattering

centers by perturbation theory. This function can be set in the following form:

$$\psi_{\vec{p}} = \psi_{\vec{p}}^{(0)} + \psi_{\vec{p}}^{(1)}, \quad (6)$$

where  $\psi_{\vec{p}}^{(1)}$  satisfies the equation:

$$i\hbar \frac{\partial}{\partial t} \psi_{\vec{p}}^{(1)} - \hat{H}^{(0)} \psi_{\vec{p}}^{(1)} = \sum_{i=1}^N U(\vec{r} - \vec{R}_i) \psi_{\vec{p}}^{(0)}. \quad (7)$$

We shall write the solution of Eq. (7) as the expansion in functions (5):

$$\psi_{\vec{p}}^{(1)} = \sum_{\vec{p}'} C(\vec{p}, \vec{p}'; t) \psi_{\vec{p}'}^{(0)}. \quad (8)$$

By substituting  $\psi_{\vec{p}}^{(1)}$  in Eq. (7), multiplying both sides of that equation by  $\Psi_{\vec{p}}^{(0)*}$ , and integrating over  $\vec{r}$ , we obtain

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} C(\vec{p}, \vec{p}'; t) &= \frac{1}{V} \int d\vec{r} \exp \left( \frac{i}{\hbar} (\vec{p} - \vec{p}') \vec{r} \right) \times \\ &\times \sum_{j=1}^N U(\vec{r} - \vec{R}_j) \exp \left\{ -\frac{i}{\hbar} (\varepsilon_{\vec{p}} - \varepsilon_{\vec{p}'} t) + \right. \\ &\left. + \frac{ie_0}{\hbar\omega c} \sum_{\alpha=1}^3 A_{\alpha}^{(0)} \left( \frac{p_{\alpha} - p'_{\alpha}}{m_{\alpha}} \right) \sin \omega t \right\}. \end{aligned} \quad (9)$$

Integrating both sides of Eq. (9) between 0 and  $t$  and taking into account the identity

$$e^{-i\lambda \sin \omega t} = \sum_{l=-\infty}^{\infty} I_l(\lambda) e^{-il\omega t} \quad (10)$$

(with  $I_l(\lambda)$  being the Bessel function), we obtain:

$$\begin{aligned} C(\vec{p}, \vec{p}'; t) &= \frac{1}{i\hbar V} \int d\vec{r} \exp \left( \frac{i}{\hbar} (\vec{p} - \vec{p}') \vec{r} \right) \times \\ &\times \sum_{j=1}^N U(\vec{r} - \vec{R}_j) \sum_{l=-\infty}^{\infty} I_l \left( \frac{e_0}{\hbar\omega c} \sum_{\alpha=1}^3 A_{\alpha}^{(0)} \frac{p_{\alpha} - p'_{\alpha}}{m_{\alpha}} \right) \times \\ &\times \frac{\exp \left\{ \left[ -\frac{i}{\hbar} (\varepsilon_{\vec{p}} - \varepsilon_{\vec{p}'} t) + il\omega \right] t \right\} - 1}{-\frac{i}{\hbar} (\varepsilon_{\vec{p}} - \varepsilon_{\vec{p}'} t) + il\omega}. \end{aligned} \quad (11)$$

Using Eq. (11), one may find the probability for an electron to pass from the state  $\vec{p}$  into the state  $\vec{p}'$  in a unit time, as a result of scattering by an impurity in the field of an electromagnetic wave:

$$P_{\vec{p}, \vec{p}'} = \frac{d}{dt} \left| C(\vec{p}, \vec{p}'; t) \right|^2. \quad (12)$$

Substituting relation (11) in Eq. (12), we obtain:

$$\begin{aligned}
 P_{\vec{p}, \vec{p}'} &= \frac{1}{\hbar^2 V^2} \left| \int d\vec{r} \exp \left( \frac{i}{\hbar} (\vec{p} - \vec{p}') \cdot \vec{r} \right) \times \right. \\
 &\times \sum_{j=1}^N U(\vec{r} - \vec{R}_j) \Big|^2 \times \\
 &\times \left\{ 2 \sum_{l=-\infty}^{\infty} I_l^2 \left( \frac{e_0}{\hbar \omega c} \sum_{\alpha=1}^3 A_{\alpha}^{(0)} \frac{(p_{\alpha} - p'_{\alpha})}{m_{\alpha}} \right) \times \right. \\
 &\times \left. \frac{\sin(\Omega - l\omega)t}{\Omega - l\omega} + \sum_{l \neq l'} (\dots) \right\}. \quad (13)
 \end{aligned}$$

In Eq. (13),  $\Omega \equiv \frac{1}{\hbar} (\varepsilon_{\vec{p}} - \varepsilon_{\vec{p}'})$ .

The terms with  $l \neq l'$  are not written explicitly in Eq. (13) since they do not contain resonance multipliers and therefore disappear at  $t \rightarrow \infty$ . Passing to the limit  $t \rightarrow \infty$  in Eq. (13) and taking into account that  $\frac{\sin xt}{x} \rightarrow \pi \delta(x)$  in this case, we obtain the following expression for  $P_{\vec{p}, \vec{p}'}$ :

$$\begin{aligned}
 P_{\vec{p}, \vec{p}'} &= \frac{2\pi}{\hbar V^2} \left| \int d\vec{r} \exp \left( \frac{i}{\hbar} (\vec{p} - \vec{p}') \cdot \vec{r} \right) \sum_{j=1}^N U(\vec{r} - \vec{R}_j) \right|^2 \times \\
 &\times \sum_{l=-\infty}^{\infty} I_l^2 \left( \frac{e_0}{\hbar \omega c} \sum_{\alpha=1}^3 A_{\alpha}^{(0)} \frac{(p_{\alpha} - p'_{\alpha})}{m_{\alpha}} \right) \times \\
 &\times \delta(\varepsilon_{\vec{p}} - \varepsilon_{\vec{p}'} - l\hbar\omega). \quad (14)
 \end{aligned}$$

Expression (14) depends explicitly on all the coordinates of ions  $\{\vec{R}_j\}$ . That is why it should be averaged over all the possible ion configurations. Taking into account that

$$\begin{aligned}
 &\left| \int d\vec{r} \exp \left( \frac{i}{\hbar} (\vec{p} - \vec{p}') \cdot \vec{r} \right) \sum_{j=1}^N U(\vec{r} - \vec{R}_j) \right|^2 = \\
 &= \sum_{j=1}^N \left| \int d\vec{r} \exp \left( \frac{i}{\hbar} (\vec{p} - \vec{p}') \cdot \vec{r} \right) U(\vec{r} - \vec{R}_j) \right|^2 + \\
 &+ \sum_{j \neq j'} \int d\vec{r}_1 \exp \left( \frac{i}{\hbar} (\vec{p} - \vec{p}') \cdot \vec{r}_1 \right) U(\vec{r}_1 - \vec{R}_j) \times \\
 &\times \int d\vec{r}_2 \exp \left( -\frac{i}{\hbar} (\vec{p} - \vec{p}') \cdot \vec{r}_2 \right) U(\vec{r}_2 - \vec{R}_{j'}), \quad (15)
 \end{aligned}$$

one may show (e.g., see [2], p. 672) that the second term in Eq. (15) turns to zero, when averaging over the positions of chaotically distributed scattering centers. Therefore, the averaging results in

$$\begin{aligned}
 &\left\langle \left| \int d\vec{r} \exp \left( \frac{i}{\hbar} (\vec{p} - \vec{p}') \cdot \vec{r} \right) \sum_{j=1}^N U(\vec{r} - \vec{R}_j) \right|^2 \right\rangle = \\
 &= N \left| \int d\vec{r} \exp \left( \frac{i}{\hbar} (\vec{p} - \vec{p}') \cdot \vec{r} \right) U(\vec{r}) \right|^2 \quad (16)
 \end{aligned}$$

. In Eq. (16),  $N$  is the number of ions in volume  $V$ , i.e.

$$N = V n_a, \quad (17)$$

with  $n_a$  being the ion concentration.

Taking into account the explicit expression for  $U(\vec{r})$  according to Eq. (2), integral (16) can be easily calculated:

$$\begin{aligned}
 &\int d\vec{r} \exp \left( \frac{i}{\hbar} (\vec{p} - \vec{p}') \cdot \vec{r} \right) U(\vec{r}) = \\
 &= \frac{4\pi e_0^2}{\varepsilon_0} \left\{ \left( \frac{\vec{p} - \vec{p}'}{\hbar} \right)^2 + \frac{1}{r_D^2} \right\}^{-2}. \quad (18)
 \end{aligned}$$

The average of  $P_{\vec{p}, \vec{p}'}$  over all the ion configurations can be obtained from Eqs. (14) and (18) as

$$\begin{aligned}
 \langle P_{\vec{p}, \vec{p}'} \rangle &= \frac{(2\pi \hbar)^3}{V} \frac{4e_0^4}{\varepsilon_0^2} \frac{n_a}{\left\{ (\vec{p} - \vec{p}')^2 + \frac{\hbar^2}{r_D^2} \right\}^2} \times \\
 &\times \sum_{l=-\infty}^{\infty} I_l^2 \left( \frac{e_0}{\hbar \omega c} \sum_{\alpha=1}^3 A_{\alpha}^{(0)} \frac{(p_{\alpha} - p'_{\alpha})}{m_{\alpha}} \right) \delta(\varepsilon_{\vec{p}} - \varepsilon_{\vec{p}'} - l\hbar\omega). \quad (19)
 \end{aligned}$$

The integral of collisions of electrons with ions in the presence of electromagnetic field looks as follows:

$$\left( \frac{\partial f}{\partial t} \right)_{\text{coll}} = - \sum_{\vec{p}'} \langle P_{\vec{p}\vec{p}'} \rangle f(\vec{p}) + \sum_{\vec{p}'} \langle P_{\vec{p}_1 \vec{p}'} \rangle f(\vec{p}'). \quad (20)$$

In Eq. (20),  $f(\vec{p})$  is the distribution function of electrons over their momentums  $\vec{p}$ . Since we consider the multivalley semiconductors, the distribution function can be different in different valleys. We further write  $f^{(i)}(\vec{p})$ , meaning the distribution function in the  $i$ th valley.

## 2. Light Absorption under Non-isotropic Impurity Scattering

We substitute now Eq. (19) in Eq. (20) and proceed from the summation over  $\vec{p}'$  to the integration. This results in the following form of the collision integral for the  $i$ th valley:

$$\begin{aligned} \left( \frac{\partial f^{(i)}}{\partial t} \right)_{\text{coll}} = & \\ = \frac{4e_0^4}{\varepsilon_0^2} n_a \sum_{l=-\infty}^{\infty} \int d\vec{p}' \frac{f^{(i)}(\vec{p}') - f^{(i)}(\vec{p})}{\left\{ (\vec{p} - \vec{p}')^2 + (\hbar/r_D)^2 \right\}^2} \times & \\ \times I_l^2 \left( \frac{e_0}{\hbar\omega c} \sum_{\alpha=1}^3 A_{\alpha}^{(0)} \frac{p_{\alpha} - p'_{\alpha}}{m_{\alpha}} \right) \delta(\varepsilon_{\vec{p}} - \varepsilon_{\vec{p}'} - l\hbar\omega). \end{aligned} \quad (21)$$

We assume that the distribution function  $f_{(\vec{p})}^{(i)}$  is normalized to the concentration in the  $i$ th valley  $n_i$ :

$$\int d\vec{p} f^{(i)}(\vec{p}) = n_i. \quad (22)$$

In thermal equilibrium, all  $n_i$  are the same. Under heating of electrons or a unidirectional pressure applied, the filling of different valleys can be also different.

We obtain the energy absorbed by an electron of the  $i$ th valley in a unit time from Eq. (21) after multiplying it by  $\varepsilon_{\vec{p}}$  and integrating over  $\vec{p}$ . At this, if we make substitutions  $\vec{p} \Leftrightarrow \vec{p}'$  and  $l \Leftrightarrow -l$  in the term which is proportional to  $f^{(i)}(\vec{p}')$  as well as define  $\varepsilon_{\vec{p}'}$  through  $\varepsilon_{\vec{p}}$  using the  $\delta$ -function, we obtain

$$\begin{aligned} P^{(i)} \equiv \int d\vec{p} \varepsilon_{\vec{p}} \left( \frac{\partial f^{(i)}}{\partial t} \right)_{st} = & \\ = -\frac{4e_0^4}{\varepsilon_0^2} n_{\alpha} \sum_{l=-\infty}^{\infty} \hbar\omega l \int \frac{d\vec{p} d\vec{p}' f^{(i)}(\vec{p})}{\left\{ (\vec{p} - \vec{p}')^2 + (\hbar/r_D)^2 \right\}^2} \times & \\ \times I_l^2 \left( \frac{e_0}{\hbar\omega c} \sum_{\alpha=1}^3 A_{\alpha}^{(0)} \frac{p_{\alpha} - p'_{\alpha}}{m_{\alpha}} \right) \delta(\varepsilon_{\vec{p}} - \varepsilon_{\vec{p}'} - l\hbar\omega). \end{aligned} \quad (23)$$

From now on, we consider only one-phonon transitions, i.e.  $l = \pm 1$ .

We have in this approximation:

$$P^i = P^{(i)}(+)-P^{(i)}(-), \quad (24)$$

where

$$\begin{aligned} P^{(i)}(\pm) = \pm \frac{4e_0^4}{\varepsilon_0^2} n \hbar\omega \int \frac{d\vec{p} d\vec{p}' f^{(i)}(\vec{p})}{\left\{ (\vec{p} - \vec{p}')^2 + (\hbar/r_D)^2 \right\}^2} \times & \\ \times I_1^2 \left( \frac{e_0}{\hbar\omega c} \sum_{\alpha=1}^3 A_{\alpha}^{(0)} \frac{p_{\alpha} - p'_{\alpha}}{m_{\alpha}} \right) \delta(\varepsilon_{\vec{p}} - \varepsilon_{\vec{p}'} \pm \hbar\omega). \end{aligned} \quad (25)$$

The sign (+) means an increase of the electron system energy (i.e., absorption), while the sign (−) means a decrease of this energy (i.e., emission).

As the estimations made for all frequencies of the optical range show, the argument of the function  $I_1(\dots)$  in Eq. (25) is far below unity. That is why we can consider only the first term of the Taylor series of  $I_1(\dots)$  in Eq. (25). We then have

$$\begin{aligned} P^{(i)}(\pm) \cong \pm \frac{e_0^6 n_{\alpha}}{\varepsilon_0^2 c^2 \hbar\omega} \int \frac{d\vec{p} d\vec{p}' f^{(i)}(\vec{p}) \delta\left\{ \varepsilon_{\vec{p}} - \varepsilon_{\vec{p}'} \pm \hbar\omega \right\}}{\left\{ (\vec{p} - \vec{p}')^2 + (\hbar/r_D)^2 \right\}^2} \times & \\ \times \left( \sum_{\alpha=1}^3 A_{\alpha}^0 \frac{p_{\alpha} - p'_{\alpha}}{m_{\alpha}} \right)^2. \end{aligned} \quad (26)$$

The distribution function must be specified to calculate integral (26). To be able to analyze further the general case, we assume that concentrations ( $n_i$ ) and temperatures ( $\theta_i$ ) in different valleys can be different.

We assume that

$$f^{(i)}(\vec{p}) = \frac{n_i}{(2\pi\theta_i)^{3/2} m_{\perp} \sqrt{m_{\parallel}}} \exp\left(-\frac{\varepsilon_{\vec{p}}}{\theta_i}\right). \quad (27)$$

In the principal axes of the mass tensor,

$$\varepsilon_{\vec{p}} = \frac{p_{\perp}^2}{2m_{\perp}} + \frac{p_{\parallel}^2}{2m_{\parallel}}. \quad (28)$$

In addition,

$$\sum_{\alpha=1}^3 A_{\alpha}^{(0)} \frac{p_{\alpha} - p'_{\alpha}}{m_{\alpha}} = \frac{\hbar\gamma}{m_{\perp}}, \quad (29)$$

where

$$\begin{aligned} \gamma \equiv \vec{A}_{\perp}^{(0)} \vec{q}_{\perp} + \frac{m_{\perp}}{m_{\parallel}} A_{\parallel}^{(0)} q_{\parallel} = & \\ = \vec{A}^{(0)} \vec{q} + \left( \frac{m_{\perp}}{m_{\parallel}} - 1 \right) \left( \vec{A}^{(0)} \vec{q} \right) \left( \vec{i}_0 \vec{q} \right), \end{aligned}$$

$$\hbar\vec{q} \equiv \vec{p} - \vec{p}'. \quad (30)$$

In Eq. (30),  $\vec{i}_0$  is the ort which specifies the direction of the line of rotation of the mass ellipsoid. The direction

of this ort coincides with that of the position of the  $i$ th valley in the laboratory system.

The angular dependence of the energy makes the integration much more difficult. That is why it is convenient to use a deformed coordinate system, in which the surfaces of equal energy are spherical.

We introduce new variables therefore:

$$\begin{aligned} p_{\perp}^* &= p_{\perp}, p_{\parallel}^* = \left(\frac{m_{\perp}}{m_{\parallel}}\right)^{1/2} p_{\parallel}; \quad q_{\perp}^* = q_{\perp}; \\ q_{\parallel}^* &= \left(\frac{m_{\perp}}{m_{\parallel}}\right)^{1/2} q_{\parallel}. \end{aligned} \quad (31)$$

At this,

$$\varepsilon_{\vec{p}} = (p^*)^2 / 2m_{\perp}. \quad (32)$$

Expression (26) acquires the following form in new variables:

$$\begin{aligned} P_{(\pm)}^{(i)} &= \pm \frac{e_0^6 n_a}{\varepsilon_0^2 c^2 \omega} \frac{m_{\parallel}}{m_{\perp}} \times \\ &\times \int \frac{d\vec{p}^* d\vec{q}^* f^{(i)}(\varepsilon_{p^*}) \delta \left\{ \frac{(\hbar q^*)^2}{2m_{\perp}} - \frac{\hbar}{m_{\perp}} p^* q^* \cos \nu^* \pm \hbar \omega \right\} \gamma^2}{\left\{ q_{\perp}^{*2} + \frac{m_{\parallel}}{m_{\perp}} q_{\parallel}^{*2} + (1/r_D)^2 \right\}^2}. \end{aligned} \quad (33)$$

Now we take into account that  $d\vec{p}^* \rightarrow p^{*2} dp^* d\Omega_{p^*} \rightarrow p^{*2} dp^* \sin \nu^* d\nu^* d\varphi^*$ .

The integral over  $\varphi^*$  can be easily calculated (since nothing depends on  $\varphi^*$ ). The integral over  $\nu^*$  can be calculated using the  $\delta$ -function:

$$\int_0^{\pi} d\nu^* \sin \nu^* \delta \left\{ \frac{(\hbar q^*)^2}{2m_{\perp}} - \frac{\hbar}{m_{\perp}} p^* q^* \cos \nu^* \pm \hbar \omega \right\} = \frac{m_{\perp}}{\hbar p^* q^*}. \quad (34)$$

Equality (34) is real under the condition that

$$|\cos \nu^*| = \left| \pm \hbar \omega - \frac{(\hbar q^*)^2}{2m_{\perp}} \right| \left/ \frac{\hbar q^* p^*}{m_{\perp}} \right. \leq 1. \quad (35)$$

Condition (35) means that the argument of the  $\delta$ -function at a specified  $q^*$  can be equal to zero. In other words, inequality (35) determines the limits of the integration over  $q^*$ . We find from Eq. (35):

$$\hbar q_{\max}(\pm) = p^* + \sqrt{p^{*2} \pm 2m_{\perp} \hbar \omega},$$

$$\hbar q_{\min}(\pm) = \left| -p^* + \sqrt{p^{*2} \pm 2m_{\perp} \hbar \omega} \right|. \quad (36)$$

After the integration over the angles of the vector  $\vec{p}^*$ , we find from Eq. (33):

$$\begin{aligned} P^{(i)}(+)&= \frac{e_0^6 n_a n_i \sqrt{m_{\parallel}}}{\sqrt{2\pi} \theta_i^{3/2} \varepsilon_0^2 c^2 \hbar \omega} \int_0^{\infty} d\varepsilon e^{-\varepsilon/\theta_i} \int_{q_{\min}^{(+)} }^{q_{\max}^{(+)}} dq^* q^* \times \\ &\times \int \frac{d\Omega_{q^*} \gamma^2(\vec{q}^*)}{\left\{ q^{*2} + \frac{m_{\parallel}}{m_{\perp}} q_{\parallel}^{*2} + (1/r_D)^2 \right\}^2}. \end{aligned} \quad (37)$$

Unlike Eq. (37), the integral over  $\varepsilon$  for the  $P^{(i)}(-)$  quantity must be calculated within limits from  $\hbar \omega$  to  $\infty$  (since only electrons with energy  $\varepsilon \geq \hbar \omega$  can emit  $\hbar \omega$  quanta). If we make a shift  $\varepsilon \rightarrow \varepsilon - \hbar \omega$  in the expression for  $P^{(i)}(-)$ , we obtain that

$$P^{(i)}(-) = -\exp\left(-\frac{\hbar \omega}{\theta_i}\right) P^{(i)}(+).$$

The latter integral over the angles of the vector  $\vec{q}^*$  in Eq. (37) can be easily calculated with a result

$$\begin{aligned} y(q^*) &\equiv \int \frac{d\Omega_{q^*} \gamma^2(\vec{q}^*)}{\left\{ q^{*2} + \frac{m_{\parallel}}{m_{\perp}} q_{\parallel}^{*2} + (1/r_D)^2 \right\}^2} = \\ &= \frac{\pi}{q^{*2}} \left\{ \left( A_{\perp}^{(0)} \right)^2 B_1(q^*) + 2 \left( A_{\parallel}^{(0)} \right)^2 \frac{m_{\perp}}{m_{\parallel}} B_2(q^*) \right\} \times \\ &\times \left( \frac{m_{\perp}}{m_{\parallel} - m_{\perp}} \right)^2. \end{aligned} \quad (38)$$

We have made the following designations in Eq. (38):

$$\begin{aligned} B_1(q^*) &= \frac{1}{b^2} + \frac{1-b^2}{b^3} \arctg \frac{1}{b}, \\ B_2(q^*) &= -\frac{1}{1+b^2} + \frac{1}{b} \arctg \frac{1}{b}, \\ b^2 &= \frac{m_{\perp}}{m_{\parallel} - m_{\perp}} \left( 1 + 1/(q^* r_D)^2 \right). \end{aligned} \quad (39)$$

The double integral in Eq. (37) can be reduced to a single one by integration by parts:

$$\begin{aligned} &\int_0^{\infty} d\varepsilon e^{-\varepsilon/\theta_i} \int_{q_{\min}^{(-)}}^{q_{\max}^{(+)}} dq^* q^* y(q^*) = \\ &= \theta_i \int_0^{\infty} d\varepsilon e^{-\varepsilon/\theta_i} \left\{ [q^* y(q)]_{q=q_{\max}^{(+)}} \frac{dq_{\max}^{(+)}}{d\varepsilon} - \right. \end{aligned}$$

$$- [q^* y(q)]_{q=q_{\min}(+)} \frac{dq_{\min}(+)}{d\varepsilon} \Big\}. \quad (40)$$

We substitute now Eqs. (40) and (38) in Eq. (37) and introduce a dimensionless variable  $x = \varepsilon/\theta_i$ . In addition, we rewrite the quantities  $A_{\perp}^{(0)}$  and  $A_{\parallel}^{(0)}$  in the laboratory coordinate system, which are written in Eq. (42) in principal axes of the  $i$ th ellipsoid (valley). This means that

$$\begin{aligned} (A_{\parallel}^{(0)})^2 &= (\vec{i}_0 \vec{q}_0)^2 A^{(0)2}, \\ (A_{\perp}^{(0)})^2 &= A^{(0)2} - (\vec{i}_0 \vec{q}_0)^2 A^{(0)2}. \end{aligned} \quad (41)$$

In Eq. (41),  $\vec{q}_0$  is an ort, which characterizes wave polarization. The ort  $\vec{i}_0$  defines the direction of the  $i$ th valley position.

We obtain the following expression as a result of the operations made above:

$$\begin{aligned} P^{(i)}(+) &= \frac{e_0^6 n_a n_i}{4\varepsilon_0^2 c^2 \hbar \omega} \left( \frac{2\pi m_{\parallel}}{\theta_i} \right)^{1/2} \frac{A^{(0)2}}{(m_{\parallel} - m_{\perp})^2} \times \\ &\times \int_0^{\infty} \frac{dx e^{-x} [\Psi_i(q_{\max}(+)) + \Psi_i(q_{\min}(+))]}{\sqrt{x(x + \hbar\omega/\theta_i)}}. \end{aligned} \quad (42)$$

We have introduced the following designation in Eq. (42):

$$\Psi_i(q^*) = B_1(q^*) + (\vec{i}_0 \vec{q}_0)^2 \left[ -B_1(q^*) + 2 \frac{m_{\perp}}{m_{\parallel}} B_2(q^*) \right]. \quad (43)$$

In dimensionless variables in accordance with Eq. (36), the quantities  $q_{\max}(+)$  and  $q_{\min}(+)$  in Eq. (42) become as follows:

$$\begin{aligned} q_{\max}(+) &= \frac{(2m_{\perp}\theta_i)^{1/2}}{\hbar} \left[ x^{1/2} + \left( x + \frac{\hbar\omega}{\theta_i} \right)^{1/2} \right], \\ q_{\min}(+) &= \frac{(2m_{\perp}\theta_i)^{1/2}}{\hbar} \left[ -x^{1/2} + \left( x + \frac{\hbar\omega}{\theta_i} \right)^{1/2} \right]. \end{aligned} \quad (44)$$

### 3. Absorption Coefficient

We have found above the energy absorbed or emitted in a unit time. Experimentally, the adsorption coefficient is measured, which looks as follows:

$$K = \frac{\sum_i (P^{(i)}(+) + P^{(i)}(-))}{\Pi} =$$

$$= \frac{\sum_i (1 - \exp(-\frac{\hbar\omega}{\theta_i})) P^{(i)}(+)}{\Pi}. \quad (45)$$

In Eq. (45),  $\Pi$  is an electromagnetic flow, which impinges on a semiconductor:

$$\Pi = \frac{\varepsilon_0^{1/2} \omega^2}{8\pi c} A^{(0)2}. \quad (46)$$

Substituting Eqs. (46) and (42) in Eq. (45), we obtain

$$\begin{aligned} K &= \frac{(2\pi)^{3/2} e_0^6 n_a m_{\parallel}^{1/2}}{\varepsilon_0^{5/2} c (m_{\parallel} - m_{\perp})^2 \hbar \omega^3} \sum_i \frac{n_i}{\sqrt{\theta_i}} \left( 1 - \exp\left(-\frac{\hbar\omega}{\theta_i}\right) \right) \times \\ &\times \int_0^{\infty} \frac{dx e^{-x} \{\Psi_i(q_{\max}(+) + \Psi_i(q_{\min}(+))\}}{\sqrt{x(x + \hbar\omega/\theta_i)}}. \end{aligned} \quad (47)$$

Expression (47) gives the general value of the adsorption coefficient under anisotropic impurity scattering in multivalley semiconductors. Different values of the filling of valleys ( $n_i$ ) can be connected with different electron temperatures ( $\theta_i$ ) in valleys, or can be caused by the unidirectional pressure (which shifts the valleys). In a state of thermodynamic equilibrium and without unidirectional pressure, all the  $n_i$  and  $\theta_i$  values are identical. The temperatures  $\theta_i$  can become different when electrons are heated by an external electric field. They can also become different under absorption of the polarized light with sufficient intensity. That is, the polarization dependence of absorption can appear in multivalley semiconductors at sufficiently high intensities. This problem was theoretically studied in [3]. When  $n_i$  and  $\theta_i$  differ in different valleys, the balance equations of concentrations and energy should be used to find them (e.g., see [4]).

The general expression for the absorption coefficient (47) under impurity scattering can be substantially simplified in the classical ( $\hbar\omega/\theta_i \ll 1$ ) and quantum ( $\hbar\omega/\theta_i \gg 1$ ) cases. Therefore, we shall analyze both these cases.

**R e g i o n o f c l a s s i c a l a b s o r p t i o n** ( $\hbar\omega/\theta_i \ll 1$ ). According to Eq. (44), we have

$$q_{\max}(+) \approx (2m_{\perp}\theta_i)^{1/2} \frac{2\sqrt{x}}{\hbar}, \quad q_{\min}(+) \approx 0. \quad (48)$$

In Eq. (44), the integration occurs over the dimensionless energy  $x = \varepsilon/\theta_i$ . Then we take into account that the quantity

$$b^2 = \frac{m_{\perp}}{m_{\parallel} - m_{\perp}} \left( 1 + \frac{1}{(q^* r_D)^2} \right) \equiv b_0^2 \left( 1 + \frac{1}{(q^* r_D)^2} \right) \quad (49)$$

has a very weak dependence on  $x$ , since (according to estimations)  $(q^* r_D)^2 \gg 1$ . Here,  $q^*$  is taken at average energies (i.e., at  $x \approx 1$ ).

We can see from Eqs. (43) and (39) that  $\Psi_i(q^*)$  can depend on  $x$  only due to the dependence of  $b^2$  on  $x$ . One may easily see that  $b^2 \rightarrow \infty$  in this case at  $q^* = q_{\min}(+) \approx 0$ . According to Eqs. (39) and (43), we obtain  $\Psi_i(q_{\min}(+)) \approx 0$ . We now pay attention to the  $\Psi_i(q_{\max}(+))$  value. At  $q^* = q_{\max}(+)$ , the value of  $b^2$  is almost independent of  $x$  since  $(q_{\max}(+) r_D)^2 \gg 1$  for all  $x \sim 1$ . The dependence on  $x$  becomes significant only at small  $x$ , for which  $(q_{\max}(+) r_D) \leq 1$ . We shall find the value of  $x = x_{\min}$  from the condition  $q_{\max}(+) r_D = 1$ . We obtain from Eq. (48):

$$x_{\min} = \frac{1}{8} \frac{\hbar^2}{m_{\perp} \theta_i r_D^2}. \quad (50)$$

Because of the stated above, when integrating approximately in Eq. (42), we can take out  $\Psi_i$  of the integration sign and truncate the integration over  $x$  at  $x = x_{\min}$ . Therefore, we obtain at  $\hbar\omega/\theta_i \ll 1$ :

$$\int_0^{\infty} \frac{dx e^{-x} \Psi_i(q_{\max}(+))}{\sqrt{x(x + \hbar\omega/\theta_i)}} \approx \Psi_i(\infty) \int_{x_{\min}}^{\infty} \frac{dx}{x} e^{-x}. \quad (51)$$

In Eq. (51), we have set  $q_{\max}(+) \approx \infty$ . This is justified at  $q_{\max}(+) r_D \gg 1$ , as can be seen from Eq. (49). As will be seen below, approximation (51) meets the known logarithmic approximation in the description of impurity scattering (so-called Conwell–Weisskopf approximation).

According to Eqs. (43) and (39), we have:

$$\begin{aligned} \Psi_i(\infty) &= \frac{1}{b_0^3} \left[ b_0 + (1 - b_0^2) \operatorname{arctg} \frac{1}{b_0} \right] \sin^2 \varphi_i + \\ &+ 2 \frac{m_{\perp}}{m_{\parallel}} \left[ -\frac{1}{1 + b_0^2} + \frac{1}{b_0} \operatorname{arctg} \frac{1}{b_0} \right] \cos^2 \varphi_i \end{aligned} \quad (52)$$

with  $\cos \varphi_i \equiv \vec{i}_0 \vec{q}_0$ , i. e.,  $\varphi_i$  is an angle between the line of rotation of the mass ellipsoid of the  $i$ th valley and the ort of the wave polarization  $\vec{q}_0$ .

Now we obtain a simpler form of the absorption coefficient for the classical region from the general expression (41), using approximation (51):

$$K = \frac{3\pi^{3/2}}{2} \frac{e_0^2}{\varepsilon_0^{1/2}} \frac{1}{c\omega^2} \sum_i n_i \left\{ \frac{\sin^2 \varphi_i}{m_{\perp} \tau_{\perp}(\theta_i)} + \frac{\cos^2 \varphi_i}{m_{\parallel} \tau_{\parallel}(\theta_i)} \right\}. \quad (53)$$

In expression (53),  $\tau_{\perp}(\theta_i)$  and  $\tau_{\parallel}(\theta_i)$  are the respective components of the relaxation tensor under impurity scattering. At this

$$\begin{aligned} \frac{1}{\tau_{\perp}(\theta_i)} &= \frac{8}{3} \frac{e_0^4 (2m_{\parallel})^{1/2}}{\varepsilon_0^2 m_{\perp} \theta_i^{3/2}} \times \\ &\times n_a \frac{b_0}{2} \left[ b_0 + (1 - b_0^2) \operatorname{arctg} \frac{1}{b_0} \right] \ln(C_1 x_{\min})^{-1}, \\ \frac{1}{\tau_{\parallel}(\theta_i)} &= \frac{8}{3} \frac{e_0^4 (2m_{\parallel})^{1/2}}{\varepsilon_0^2 m_{\parallel} \theta_i^{3/2}} \times \\ &\times n_a b_0 \left[ -b_0 + (1 + b_0^2) \operatorname{arctg} \frac{1}{b_0} \right] \ln(C_1 x_{\min})^{-1}. \end{aligned} \quad (54)$$

$\ln(C_1 x_{\min})^{-1}$  appears in Eqs. (54) because the integral on the right part of Eq. (51) is equal to

$$\int_{x_{\min}}^{\infty} \frac{dx}{x} e^{-x} = \ln(C_1 x_{\min})^{-1} - \sum_{k=1}^{\infty} \frac{1}{k!} (-x_{\min})^k,$$

where  $\ln C_1 = 0.577\dots$  is the Euler constant. Since  $x_{\min} \ll 1$ , we have confined ourselves within only the logarithmic approximation in Eq. (54).

The components of the relaxation tensor under impurity scattering are connected with the mobility tensor components by the following relations:

$$\mu_{\perp} = \frac{8}{\sqrt{\pi}} \frac{e_0 \tau_{\perp}(\theta_i)}{m_{\perp}}; \quad \mu_{\parallel} = \frac{8}{\sqrt{\pi}} \frac{e_0 \tau_{\parallel}(\theta_i)}{m_{\parallel}}. \quad (55)$$

**R e g i o n o f q u a n t u m a b s o r p t i o n**  
( $\hbar\omega \gg \theta_i$ ).

In this case, we obtain from Eq. (44):

$$q_{\max}(+) \approx q_{\min}(+) = \left( \frac{2m_{\perp}}{\hbar} \omega \right)^{1/2} \equiv q_{\omega}. \quad (56)$$

Now integral (47) can be easily estimated:

$$\begin{aligned} \int_0^{\infty} \frac{dx e^{-x} \{ \Psi_i(q_{\max}(+)) + \Psi_i(q_{\min}(+)) \}}{\sqrt{x(x + \hbar\omega/\theta_i)}} &\approx \\ &\approx 2\sqrt{\pi} \Psi_i(q_{\omega}) \left( \frac{\theta_i}{\hbar\omega} \right)^{1/2}. \end{aligned} \quad (57)$$

We may set  $\Psi_i(q_{\omega}) \approx \Psi_i(\infty)$ , taking into account that  $(q_{\omega} r_D) \gg 1$ . As a result, for the quantum region ( $\hbar\omega \gg \theta_i$ ), we have from Eq. (41):

$$K = \left( \frac{2\pi}{\varepsilon_0} \right)^{5/2} \frac{e_0^6 n_a m_{\parallel}^{1/2}}{c(m_{\parallel} - m_{\perp})^2 \omega^2} \frac{\sum_i n_i \Psi_i(\infty)}{(\hbar\omega)^{3/2}}. \quad (58)$$

The form of the function  $\Psi_i(\infty)$  is specified by formula (52), where the explicit dependence on the polarization angle  $\varphi_i$  is given.

Thus, we have obtained a simple expression for the light adsorption coefficient in the classical and quantum ranges of frequencies under the dominating role of impurity (anisotropic) scattering.

To complete the picture, we also present the expression for the adsorption coefficient in the case of the dominating role of acoustic scattering. This will enable us to compare the peculiarities of manifestation of various scattering mechanisms in the phenomenon of light absorption by free carriers.

The absorption coefficient in the case of anisotropic acoustic scattering was obtained in [1]. Before writing it down, we recall (e.g., see [1]) that the components of the relaxation tensor as functions of electron energy in semiconductors of the kind of  $n$ -Ge and  $n$ -Si can be written in the following form under acoustic scattering:

$$\begin{aligned}\tau_x(\varepsilon) = \tau_y(\varepsilon) &\equiv \tau_{\perp}(\varepsilon) = \tau_{\perp}^{(0)} \left( \frac{\theta_i}{\varepsilon} \right)^{1/2}; \\ \tau_{\parallel}(\varepsilon) &= \tau_{\parallel}^{(0)} \left( \frac{\theta_i}{\varepsilon} \right)^{1/2}.\end{aligned}\quad (59)$$

The general expression for the adsorption coefficient in the whole frequency range under anisotropic acoustic scattering has a following form [1]:

$$\begin{aligned}K = & -\frac{16\sqrt{\pi}}{3\sqrt{\varepsilon_0}} \frac{e_0^2}{c\hbar} \sum_i \frac{n_i \theta_i}{\omega^3} (1 - e^{-\hbar\omega/\theta_i}) \times \\ & \times \left\{ \frac{\sin^2 \varphi_i}{m_{\perp} \tau_{\perp}(\theta_i)} + \frac{\cos^2 \varphi_i}{m_{\parallel} \tau_{\parallel}(\theta_i)} \right\} \left\{ a_i^3 \frac{d}{da_i} \left( \frac{K_1(a_i)}{a_i} \right) \right\}.\end{aligned}\quad (60)$$

[In Eq. (60), we have corrected the misprints made in this formula in [1].]

In Eq. (60),  $a_i = \hbar\omega/2\theta_i$  and  $K_1(a_i)$  is a Bessel function, whose asymptotic form is as follows:

$$K_1(x) = \begin{cases} \frac{1}{x} & \text{at } x \rightarrow 0, \\ \sqrt{\frac{\pi}{2x}} e^{-x} & \text{at } x \rightarrow \infty. \end{cases}\quad (61)$$

To avoid misunderstanding, we notice that  $\tau_{\perp,\parallel}^{(0)}$  in Eq. (59) differs from  $\tau_{\perp,\parallel}^{(0)}$  in [1] by a factor  $(\frac{\theta_i}{\theta})^{1/2}$ , since we have  $\tau_{\perp,\parallel}(\theta) = (\frac{\theta_i}{\theta})^{1/2} \tau_{\perp,\parallel}^{(0)}$  from Eq. (59), while  $\tau_{\perp,\parallel}(\theta) = \tau_{\perp,\parallel}^{(0)}$  in [1]. We do not introduce the

lattice temperature  $\theta$  at all in this paper. The electron temperatures  $\theta_i$  stand everywhere, which can coincide with the lattice temperature or differ from it.

We obtain the absorption coefficient in the classical and quantum ranges of frequencies from the general formula (60), using asymptotics (61).

Therefore, we get in the classical range ( $\hbar\omega \ll \theta_i$ ):

$$K = \frac{32\sqrt{\pi}}{3} \frac{e_0^2}{\sqrt{\varepsilon_0}} \frac{1}{c\omega^2} \sum_i n_i \left\{ \frac{\sin^2 \varphi_i}{m_{\perp} \tau_{\perp}(\theta_i)} + \frac{\cos^2 \varphi_i}{m_{\parallel} \tau_{\parallel}(\theta_i)} \right\}.\quad (62)$$

In the quantum range ( $\hbar\omega \gg \theta_i$ ), we have, respectively,

$$\begin{aligned}K = & \frac{4\pi}{3} \frac{e_0^2}{\sqrt{\varepsilon_0}} \frac{1}{c\omega^2} \sum_i n_i \left( \frac{\hbar\omega}{\theta_i} \right)^{1/2} \times \\ & \times \left\{ \frac{\sin^2 \varphi_i}{m_{\perp} \tau_{\perp}(\theta_i)} + \frac{\cos^2 \varphi_i}{m_{\parallel} \tau_{\parallel}(\theta_i)} \right\}.\end{aligned}\quad (63)$$

From the comparison of formulas (53) and (62), one may see that the light absorption coefficient in the classical frequency range depends equally on the components of the tensor of relaxation times and the components of the mass tensor both under impurity and acoustic scattering. The only difference is in numerical coefficients, which is stipulated by the different energy dependence of the relaxation times under impurity and acoustic scattering.

A totally different situation appears in the quantum region, as can be seen from the comparison of Eqs. (58) and (63). The reason for these differences is that the impurity scattering potential (18) would have a singularity at  $\omega \rightarrow 0$  without screening (i. e., formally at  $r_D \rightarrow \infty$ ). Therefore, the screening effect for a charged impurity should be taken into account in the classical region. This screening is not significant in the quantum frequency region. For the acoustic scattering, in difference to Eq. (55), we get

$$\mu_{\alpha} = \frac{4}{3\sqrt{\pi}} \frac{e\tau_{\alpha}(\theta_i)}{m_{\alpha}}.\quad (64)$$

#### 4. Polarization Effects under Light Emission by Free Carriers

If the electron gas is heated (e.g., by electric current), the effect opposite to the Drude absorption occurs, i.e., free carriers emit light. Polarization dependences can appear in the case of the anisotropic dispersion law of free carriers. Such polarization effects take place in the cases of different heatings or under the same heating but with different fillings of the valleys.



We can obtain the spontaneous emission by hot electrons, which is of interest for us, using the expression for the field-induced emission, which we derived previously. For this, the vector potential ( $\vec{A}^{(0)}$ ) of a wave should be first normalized so that  $N_{\text{ph}}$  photons are in volume  $V$ , i.e. the following condition should be used:

$$\frac{1}{V} N_{\text{ph}} \hbar \omega = \frac{E^2}{4\pi} = \frac{1}{8\pi} \left( \frac{\omega}{c} \right)^2 A^{(0)2}. \quad (65)$$

From here,

$$A^{(0)} = 2c \left( \frac{2\pi\hbar}{V\omega} N_{\text{ph}} \right)^{1/2}. \quad (66)$$

Then we should substitute expression (65) in the formula for  $P^{(i)}(-)$ , setting  $N_{\text{ph}} = 1$ .

And, finally, we should multiply the obtained expression by the density of finite field states in a unit frequency interval and a solid angle  $d\Omega$ :

$$d\rho(\omega) = \frac{V}{(2\pi c)^3} \omega^2 d\Omega. \quad (67)$$

As a result of procedures described, we obtain the following expression from  $P^{(i)}(-)$  for the emission of electrons in all the valleys into a solid angle  $d\Omega$  in the case of impurity scattering:

$$W^{(-)} = \frac{e_0^6 n_a \sqrt{m_{\parallel}} d\Omega}{(2\pi)^{3/2} \varepsilon_0^2 c^3 (m_{\parallel} - m_{\perp})^2} \times \sum_i \frac{n_i}{\sqrt{\theta_i}} e^{-\hbar\omega/\theta_i} \int_0^{\infty} \frac{dx e^{-x} \{ \Psi_i(q_{\text{max}}) + \Psi_i(q_{\text{min}}) \}}{\sqrt{x(x + \hbar\omega/\theta_i)}}. \quad (68)$$

Expression (68) gives the emission intensity from a unit volume with  $\sum_i n_i$  electrons. To obtain the emission from an arbitrary volume  $V$ , expression (68) should be multiplied by  $V$ . Note that the signs of the expressions  $P^{(i)}(+)$  and  $P^{(i)}(-)$  are different, since  $P^{(i)}(+)$  characterizes the energy incorporation into the electron subsystem (i.e., absorption), while  $P^{(i)}(-)$  describes the energy extraction from it. In Eq. (68), we use the absolute value of emission intensity.

One may derive simple expressions in the limiting cases of the classical and quantum frequency ranges from the general expression (68), similarly to the case of absorption.

In the case of the classical frequency range ( $\hbar\omega \ll \theta_i$ ), we have:

$$W^{(-)} = \frac{e_0^6 n_a \sqrt{m_{\parallel}}}{(2\pi)^{3/2} \varepsilon_0^2 c^3 (m_{\parallel} - m_{\perp})^2} \times$$

$$\times \sum_i \frac{n_i}{\sqrt{\theta_i}} \Psi_i(\infty) \ln(C_1 x_{\text{min}})^{-1} d\Omega. \quad (69)$$

We obtain, respectively, in the quantum frequency range ( $\hbar\omega \gg \theta_i$ ):

$$W^{(-)} = \frac{e_0^6 n_a \sqrt{m_{\parallel}}}{\sqrt{2\pi} \varepsilon_0^2 c^3 (m_{\parallel} - m_{\perp})^2} \frac{1}{\sqrt{\hbar\omega}} \times \sum_i n_i \Psi_i(\infty) e^{-\hbar\omega/\theta_i} d\Omega. \quad (70)$$

Using the explicit expression (52) for  $\Psi_i(\infty)$  and the formulas for the components of the relaxation tensor under impurity scattering (54), formula (69) becomes

$$W^{(-)} = \frac{3e_0^2}{16\pi^{3/2} c^3} \sum_i n_i \theta_i \left\{ \frac{\sin^2 \varphi_i}{m_{\perp} \tau_{\perp}(\theta_i)} + \frac{\cos^2 \varphi_i}{m_{\parallel} \tau_{\parallel}(\theta_i)} \right\} d\Omega. \quad (71)$$

The term  $\ln(C_1 x_{\text{min}})^{-1}$  in Eq. (69) is related to the screening effect of the Coulomb potential of an impurity. The screening effect is not significant in the quantum frequency range. Therefore, this term does not appear in formula (70) which cannot be expressed by the components of the relaxation tensor, as in the case of Eq. (71).

Under the dominating role of acoustic scattering, the energy emitted by all electrons in all the valleys per unit time into a solid angle  $d\Omega$  is equal to

$$W^{(-)} = \frac{-2e_0^2}{3\pi^{5/2} c^3} \sum_i n_i \theta_i e^{-\hbar\omega/\theta_i} \times \left\{ \frac{\sin^2 \varphi_i}{m_{\perp} \tau_{\perp}(\theta_i)} + \frac{\cos^2 \varphi_i}{m_{\parallel} \tau_{\parallel}(\theta_i)} \right\} a_i^3 e^{a_i} \frac{d}{da_i} (K_1(a_i)/a_i) d\Omega. \quad (72)$$

We obtain from here for the classical frequency range ( $\hbar\omega \ll \theta_i$ ):

$$W^{(-)} = \frac{4e_0^2}{3\pi^{5/2} c^3} \sum_i n_i \theta_i \left\{ \frac{\sin^2 \varphi_i}{m_{\perp} \tau_{\perp}(\theta_i)} + \frac{\cos^2 \varphi_i}{m_{\parallel} \tau_{\parallel}(\theta_i)} \right\} d\Omega. \quad (73)$$

A simple expression for the emission intensity can also be obtained from Eq. (72) in the quantum frequency range case ( $\hbar\omega \gg \theta_i$ ):

$$W^{(-)} = \frac{e_0^2}{6\pi^2 c^3} \sum_i \frac{n_i}{\sqrt{\theta_i}} (\hbar\omega)^{3/2} e^{-\hbar\omega/\theta_i} \times$$

$$\times \left\{ \frac{\sin^2 \varphi_i}{m_{\perp} \tau_{\perp}(\theta_i)} + \frac{\cos^2 \varphi_i}{m_{\parallel} \tau_{\parallel}(\theta_i)} \right\} d\Omega. \quad (74)$$

We can see from Eq. (73) that the emission intensity does not depend on the emitted light frequency in the classical frequency range, and drops exponentially in the quantum frequency range. We recall that  $\tau_{\perp}$  and  $\tau_{\parallel}$  in Eqs. (72)–(74) are the components of the acoustic relaxation tensor, which are set by formula (59).

We can see from the comparison of formulas (71) and (73) that the dependence on the parameters is the same. The numerical coefficients are only different, which is stipulated by the different dependences of the relaxation tensor components on the electron energy under impurity and acoustic scattering.

## 5. Conclusion and Remarks

In this paper, the general expressions are obtained for the absorption coefficient, as well as for the emission intensity in the presence of hot electrons. These expressions are derived with taking into account the multivalley character of the electron spectrum as well as the anisotropy of the dispersion law and the scattering mechanisms. The obtained expressions depend both on the concentration of electrons  $n_i$  and their temperatures  $\theta_i$  in individual valleys.

In the case of thermodynamic equilibrium, all  $\theta_i$  values are the same (and are equal to the lattice temperature). Moreover, the populations  $n_i$  in all valleys are also identical when the unidirectional pressure is absent. Under unidirectional pressure applied to a specimen,  $n_i$  are the known functions of the applied mechanical stress (e.g., see [5]).

Under an electric field applied, all  $\theta_i$  and  $n_i$  values (or a part of them) can be different. The procedure for their calculation is well known (e. g., see [4]).

The especially simple case of the polarization dependence of emission appears when all electrons

migrate to a single valley. It is remained the only dependence on one angle — between the polarization ort and a rotation axis of the ellipsoid of the surface of equal energy of the populated valley. Experimentally, the polarization dependences in  $n$ -Ge have been investigated in [6].

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## ОСОБЛИВОСТІ ПОГЛИНАННЯ І ВИПРОМІНЮВАННЯ СВІТЛА ВІЛЬНИМИ ЕЛЕКТРОНАМИ В БАГАТОДОЛИННИХ НАПІВПРОВІДНИКАХ

П.М. Томчук

### Резюме

Отримано загальні вирази для коефіцієнта поглинання світла вільними носіями і інтенсивності спонтанного випромінювання світла гарячими електронами в багатодолінних напівпровідниках. Отримані вирази залежать від концентрації електронів в окремих долинах і їх температур. Враховано анізотропію закону дисперсії і механізмів розсіяння електронів. Розглянуто домішковий і акустичний механізм розсіяння. Встановлено поляризаційну залежність спонтанного випромінювання гарячих електронів. У випадку однонапрявленого тиску або великих інтенсивностей опромінення поляризаційну залежність виявляє і коефіцієнт поглинання світла вільними електронами.